Chapter 5, all you ever wanted to know about Q but were afraid to ask.

Because after chapter 3 and 4 there where questions about Q, I have made some extra information about this subject. This will be more theory but there are some fun experiments involved as usual.

To begin, I like this story that comes from a Bontoon publication about Q; Ask 6 blind men to investigate an elephant, report their findings and determine the true nature of an elephant.
The one that touched the side of the beast said, it looks like a wall, the one touching the tail proclaimed an elephant was like a rope, the third chancing up a leg avowed the elephant to be a kind of a tree and so on. The conclusion: each was partly in the right and all were in the wrong. Just like this it is with the Q.

There are several ways of looking at the Q. Like selectivity, resonance rise in voltage , currents, reactance/resistance related or the envelope of a damped wave train.

Q of the coil is most times regarded as the ratio of reactance to it's **series** resistance. Q=Xl/Rs
Because the reactance is involved , the Q is also frequency depended. As you know Xl changes with frequency so Q for a coil at 10KHz of 1GHz is a big difference. At 10KHz you only have the copper resistance but a very low Xl, at 1Ghz you get other losses like skin effect loss but an immense Xl. So first Q will rise with frequency and after a certain point it will go down again.

Q is a **dimensionless** ratio, just a number. Q is not a "defined" value so. If I give you a coil and tell you it has a Q of 100 you still know nothing.

What is Q:
First two basics;

- only at AC **current** we talk about Q
- Capacitors and inductors store energy and deliver it back to the circuit
- Not all energy will be delivered back, some magically disappears.

So Q must be a figure of merit for the circuit or component we are looking at. The ratio between it's ability to store energy and the energy lost. Ideal we look at it during one cycle.

For the math geeks:

The energy lost per cycle is equal to the average power times the time of one cycle, $T \doteq (1/f)$, or $1/2 R_s I^2 T$.

The ratio of stored energy to energy dissipated per cycle becomes:

$$\frac{\frac{1}{2} L I^2}{\frac{1}{2} I^2 R_s T} = \frac{1}{T}\,\frac{L}{R_s} = \frac{fL}{R_s} = \frac{1}{2\pi}\,\frac{2\pi f L}{R_s}$$

$$= \frac{1}{2\pi}\,\frac{\omega L}{R_s} = \frac{1}{2\pi}\,Q$$

For the rest of us; Q = 2pi(total energy stored/energy dissipated per cycle) but in this case I talked about a series circuit. All losses are in the Rs in that case. But not only the Rs of the Coil. But also that of your wires, capacitor, solderjoints ect. Makes measuring a bit hard.

If we have a **parallel** circuit the Rs of the capacitor and Coil are parallel to each other. So our Rs is changed in Rp. At resonance the current flowing through the circuit is controlled by the parallel impedance. So now we have a impedance that is almost infinitive high paralleled with a tiny resistor that is formed by the resistor causing the losses. Q=Rp/(wL)

In the series case the **Q will be high if the Rs is small.** In the parallel case the **Q will be high if the total ohms Rp is high**.

For the Math addicts:

$$Z_{AB} = \frac{(-j\frac{1}{\omega C})(j\omega L + Rs)}{(-j\frac{1}{\omega C}) + (j\omega L + Rs)}$$

At resonance: $\left|\frac{1}{\omega C}\right| = |\omega L| = X$,

where $|\ |$ indicates magnitude, so that

$$Z_{AB} = \frac{(-jX)\ (+jX + Rs)}{-jX\ + jX + Rs} = \frac{X^2 - jXRs}{Rs} = \frac{X^2}{Rs} + (-jX).$$

The absolute magnitude of this impedance is

$$Z_{AB} = \sqrt{\left(\frac{X^2}{Rs}\right)^2 + X^2} = X\sqrt{\frac{X^2}{R_s^2} + 1}.$$

Or, $Z = \omega L\sqrt{Q^2 + 1}$

For most practical purposes this reduces to:

$$Z = Q\omega L,$$

Equating: $\frac{E}{Q\omega L} = \frac{E}{Rp}$; or, $Q\omega L = Rp$

Rewriting: $Q = Rp/\omega L$.

where R_p = total effective parallel circuit resistance in ohms.

Bandwidth

I hear you now thinking, but Fred, we always learned Q is about bandwidth ? That is indeed an other way to look at the elephant called Q. We have seen that it is about energy in a resonant circuit and that by looking at the current. If the circuit is not resonant the situation changes because impedance then changes rather quick. So lets look at that case. First we have to define **bandwidth**. Thankfully some people in the past have decided for us how to define that.
This is the frequency in which the circuit reactance is as large as the total circuit resistance. If two impedances or resistances are equal we have the famous 3dB point. The power is half the value it is at resonance. There is such a point below and above the resonant frequency as can be seen in the chapter about inductors. Z at that point will be 1.41421356237 x Rs
This impedance is made from a real and an imaginary part. We are interested in the R part because this part is the cause for power loss and so this decreases the Q.

If we take a LC circuit and hook it up to our generator and apply the same voltage to this frequency as we do at resonance the current at the 3dB frequency will be 0.707 times smaller than at resonance. The power dissipated will then be half as big or 3dB less as at resonance.
Don't you love it when all pieces fit ? Later I will show you how to do this.

Just some math again, you are not used to get that from me. But just because there is not much lecture on this I include it here for those who actual knows what they mean and are not suffering from numberfobia like me.

The last part in this formula is the one we all learned in the books. It is based on the power dissipated in a circuit at two selected frequencies.

$$\Delta X = 2(2\pi \Delta f L) = 4\pi \Delta f L.$$

$$R_s = 4\pi (f_0 - f_1)\ L = 4\pi f_0 L - 4\pi f_1 L$$

$$R_s = 4\pi (f_2 - f_0)\ L = -4\pi f_0\ L + 4\pi f_2 L.$$

$$2R = 4\pi(f_2 - f_1)L.$$

$$\frac{f_0}{(f_2 - f_1)} = \frac{2\pi f_0 L}{R_s} = \frac{\omega L}{R_s} = Q$$

Voltage

But There are other ways to look at Q. We can also look at the behavior of the voltage.
We take a **series** circuit this time. This has besides a capacitor and a coil also a series resistance.
In this circuit we have a current going around caused by an sinusoidal voltage we applied with a generator. At resonance the impedance is infinitive small. So the only current regulating part is the real resistance. I=u/Rs according to Ohm and because it is “Ohms law” and not “Ohms advise” we follow that here also. We know a resistor is a sort of “current into voltage” transformer. So there will be a voltage over the resistor. Because the resistance is part of the inductor we measure that voltage over the inductor. U = I x wL = (u/Rs) x wL (at resonance). This gives us yet an other formula for Q. That will be U/u = wL/Rs = Q Also written as U=Qu. The big U is the measured voltage, the small u is the sourcevoltage. If Rs is high the Q will be low and in that case we must also account for the drop across the resistor. U= u x SQRT(1+Q^2)

Damped waves:

If we use a LC circuit in an oscillator and there was no Rs or Rp but just ideal perfect parts, the energy pumped in the circuit by our powersupply would stay there for ever going round and round. But unfortunate we do not live in a perfect world so we lose a little bit of energy in each cycle. If our powersupply would not made up for this loss, the waves would become smaller and smaller each cycle upto the point the circuit ran out of energy. The phenomena describing this is called the damping coefficient ; (R/2L) . We know how long one cycle is because we know the frequency so lets multiply this with the damping coefficient. What we get is the logarithmic decrement of the circuit. Every periode (T) we get a bit of damping. But also a change in current. And Q was about currents too. So we get also changes in current every cycle. I can not type this because the delta sign so here a picture:

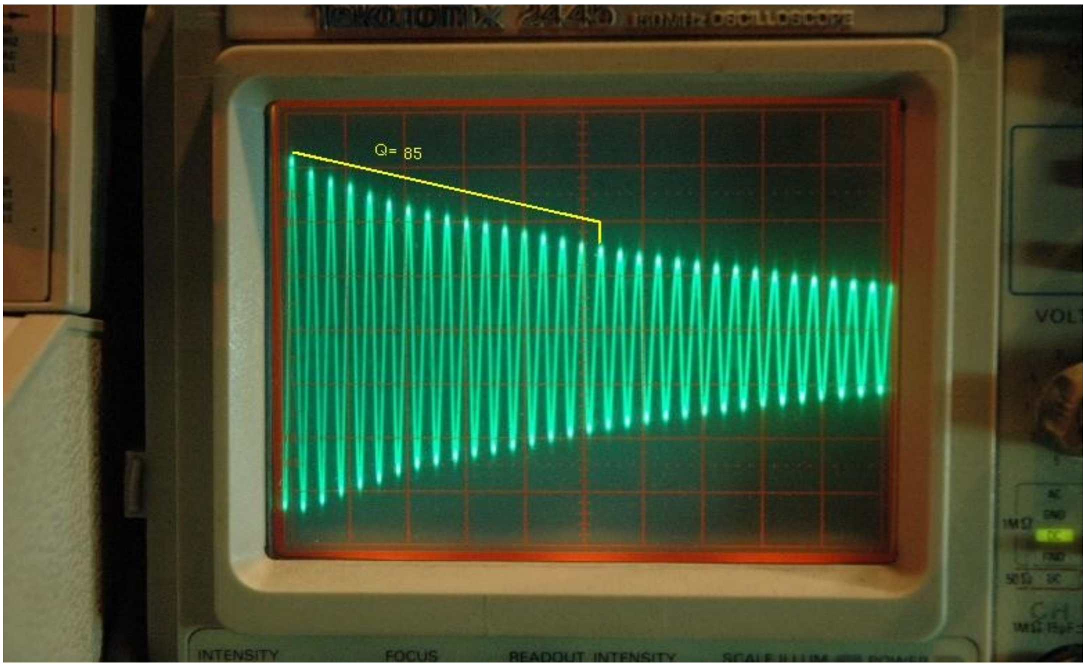
$$\frac{I_2}{I_1} = \epsilon^{-\frac{R}{2L} T} = \epsilon^{-\delta}$$

But $T = \frac{1}{f}$, so $\delta = \frac{R_s}{2fL}$, or $\delta = \frac{\pi}{Q}$

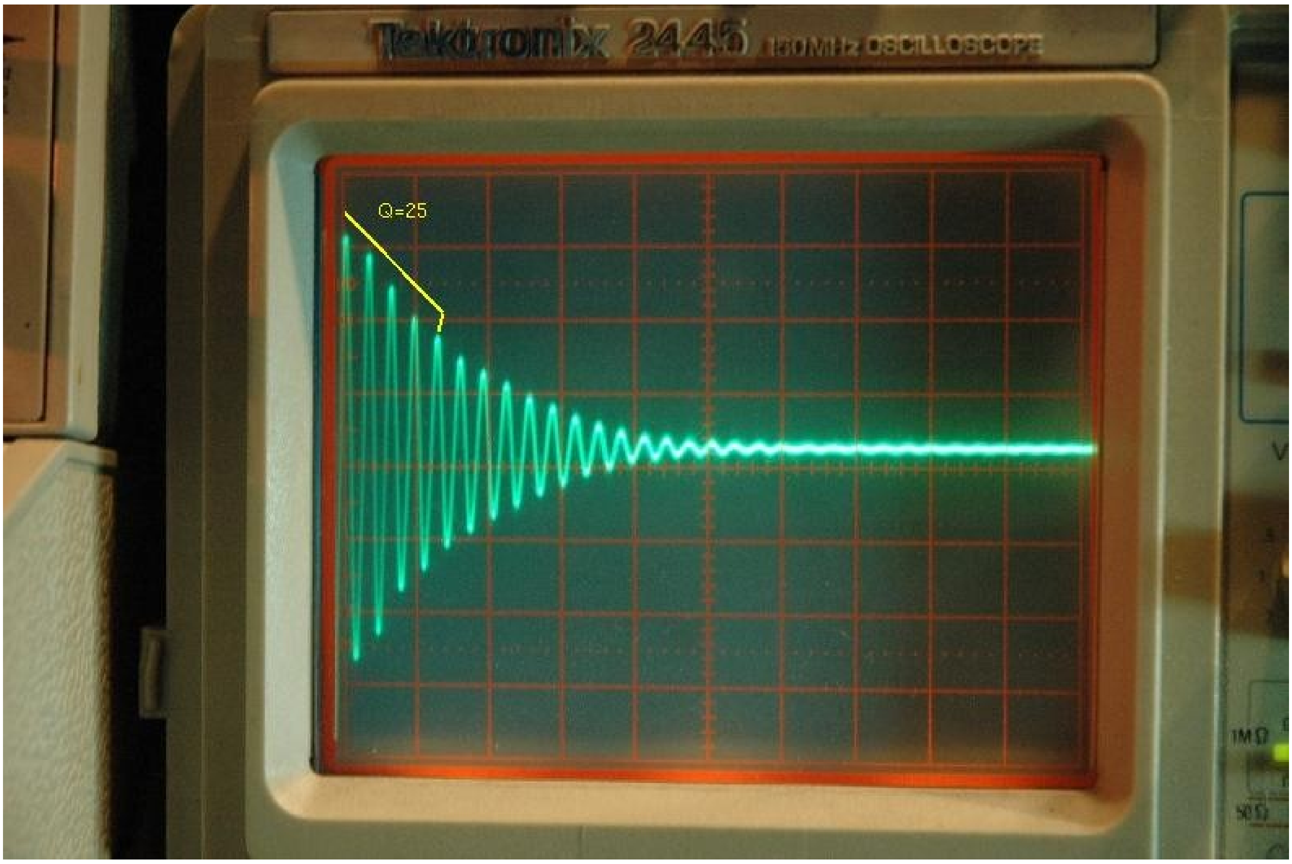
Rewriting: $Q = \frac{\pi}{\delta}$

So you see, yet an other formula of Q. Will it never stop ?

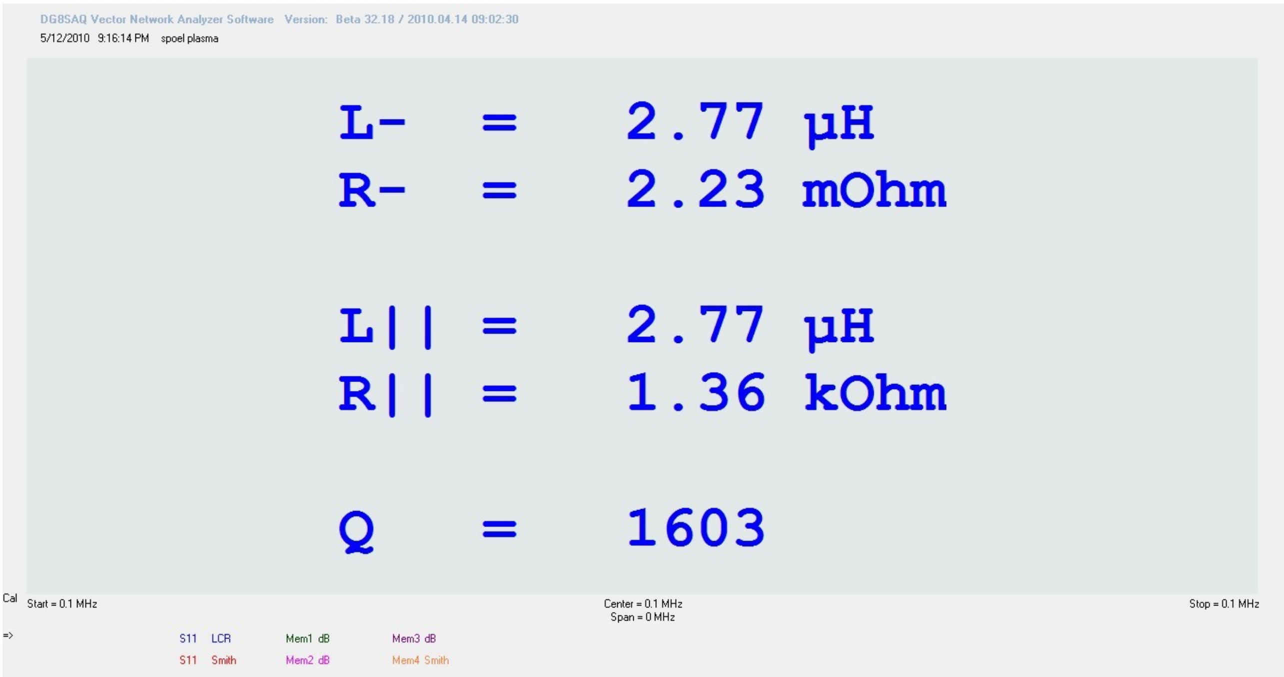
DIY Q measurement the nasty way
This is the way you often see in damped waves measurements. You count the waves upto the point the amplitude on your scope is just half of the first. The most difficult thing is the coupling because everything you include forms a part of the circuit. Our VNA can be calibrated but still influences the circuit because the coil can dump its energy in our 50 ohm resistor. But in a set up with signal generator and oscilloscope we can not calibrate anything. So we couple the signal in by means of a coil and we also pick up the damping waves by a coil. The problem is we must pick up the signals some way and what ever way we chose, that energy is coming from our circuit so we create some extra losses. (That's also how a griddipper works) So this method is rather tricky. With high Q coils I never measured more then a Q around 90. Changing the coupling or other things changes the Q. So this is not a very good way, other then to compare two coils in the same situation. But that is the problem with every Q measurement. Quantum mechanics learns us the observer is always part of the system and that is certain the fact here. (translation: inkoppelspoel stands for the coil that couples the generator to the coil and blok is a square wave at 10KHz) The other one goes to the scope. The coils both have a 50 ohm series resistance.



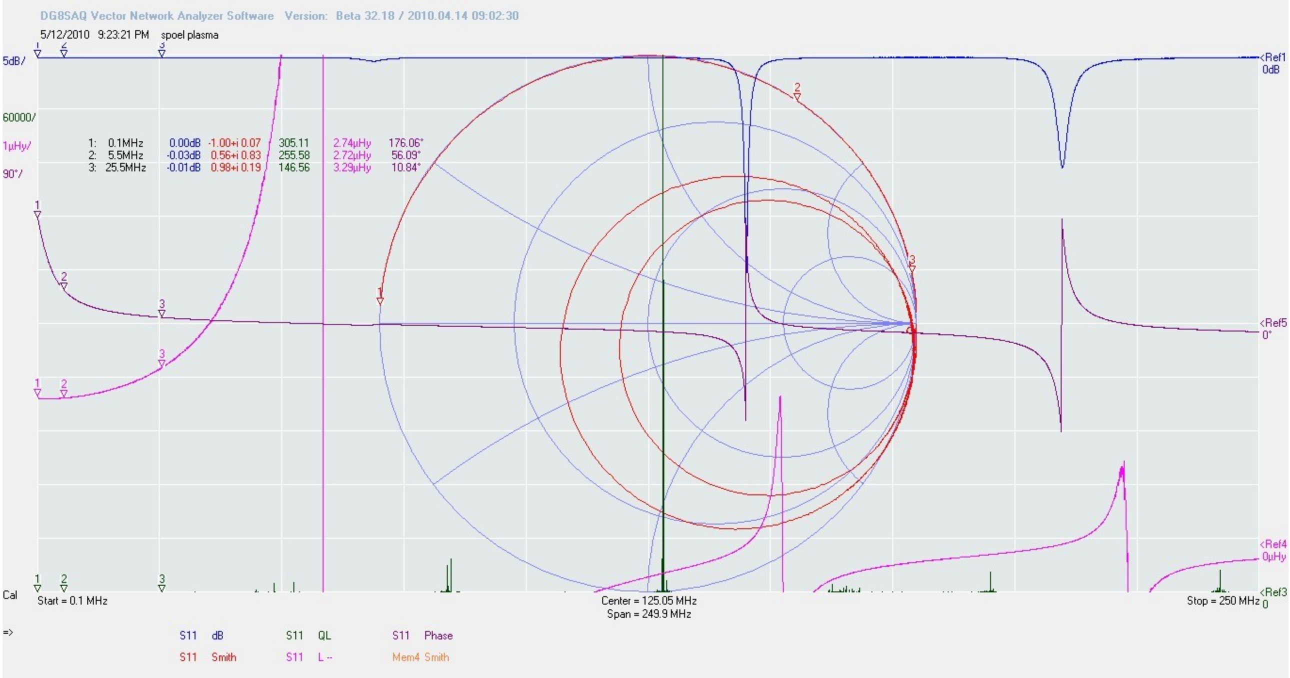
If I hook up the same coil direct to the generator and to the scope by a 10:1 probe I get this. Rest my case.



This is what the VNA makes out of this coil.



But a sweep gives yet another picture:

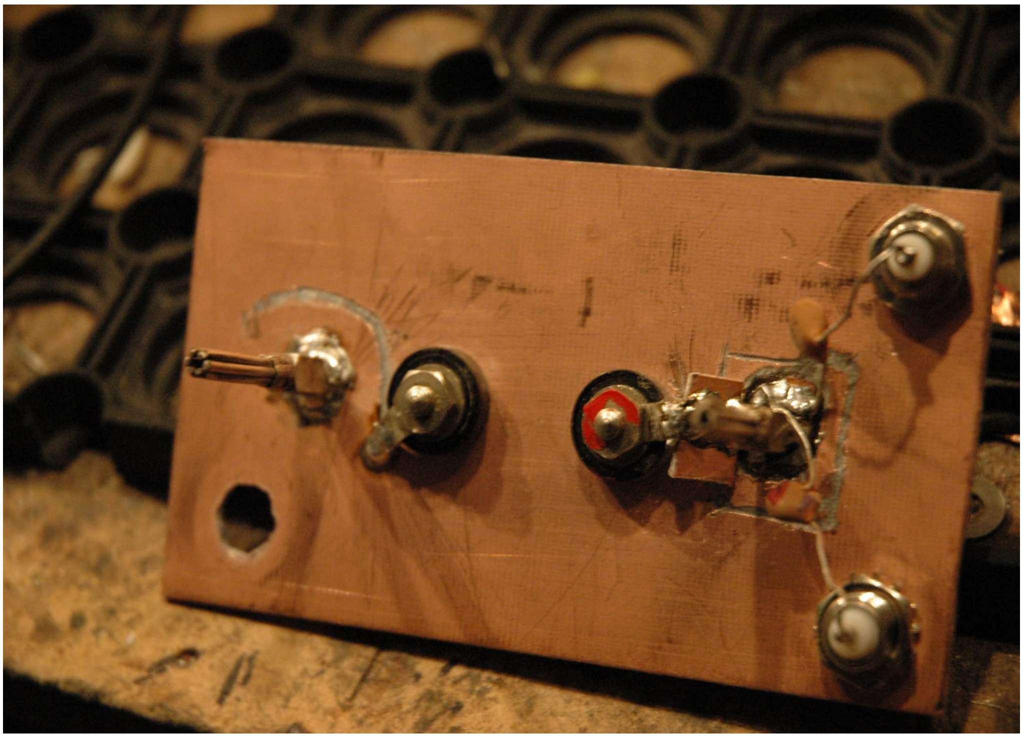


You see, frequency and the way you measure influences the Q. If you want to know the real unloaded Q you can not measure it because then you have to be invisible for the coil, to bad it will be invisible for you too. But it is great to compare a few coils in the same situation.

	voltcraft	philips	mm	mm	draad	kern	mm	mm		theorie	theorie	vorm	meet	vna	vna	vna	meet	nameting L	Q factor	blokgolf
	L gemeten	C pf	D kern	d draad	mat	materiaal	L lengte	L-draad	n=	Rskin	schijn Q	factor	F in Khz	3dB BW	L berekend	Q	F in Mhz	TF1313	TF1313	Q damped
5	2,84	450	50	1,4	vd	air	25	980	6	0,2	1243	0,48	4351,6	24	2,97	181,32	4,35	2,81	4,3	185

This is some cut/paste from an sheet I made after a Q measurement before I had a VWNA. I used a HP8407 VNA in that test. The coil in the tests above is one of my test-coils. Number 5. Very easy to have some known coils. The first two fields show the inductance measured by a digital LCR meter. The white field are about the coil (wire, size ect) Yellow is the theoretical calculated values. Green are calculated from VNA measurements. Blue is the result from my Marconi compensated LCR bridge and Dark green is the Q measured with the method of measuring the 3dB points.
The capacitor used was this calibrated beast with a special made adapter.

On the other side of that adapter are two banana jack busses to connect the coil. The generator and scope are coupled by a 1.5pF capacitor to minimize load.



The last way to look at is is by the way the phase angle changes between the driving voltage and the current in a resonant circuit. If we consider the coil as a circuit on it's own the formula for Q will be :

$$\tan \phi = \omega L / R_s = Q$$

Or: $Q =$ (tangent of the phase angle.)

Closely associated with the phase angle is the *power factor*. The power factor of an inductor is the ratio of the total *resistance* absorbing power to the total *impedance* of the device, and is designated by Cos ϕ :

$$\cos \phi = \frac{R_s}{\sqrt{R_s^2 + \omega^2 L^2}} = \frac{R_s}{R_s \sqrt{1 + \frac{\omega^2 L^2}{R_s^2}}}$$

$$= \frac{1}{\sqrt{1 + Q^2}}$$

This is approximately $\cos \phi = \frac{1}{Q}$.

This should be a way a VNA does it.

OK, there is an other option, if you still can find one. They used to make **Q meters**. If you find one you need a standard to calibrate them. Good luck :-) It was sold in a wooden box and that means it was a lot of money.

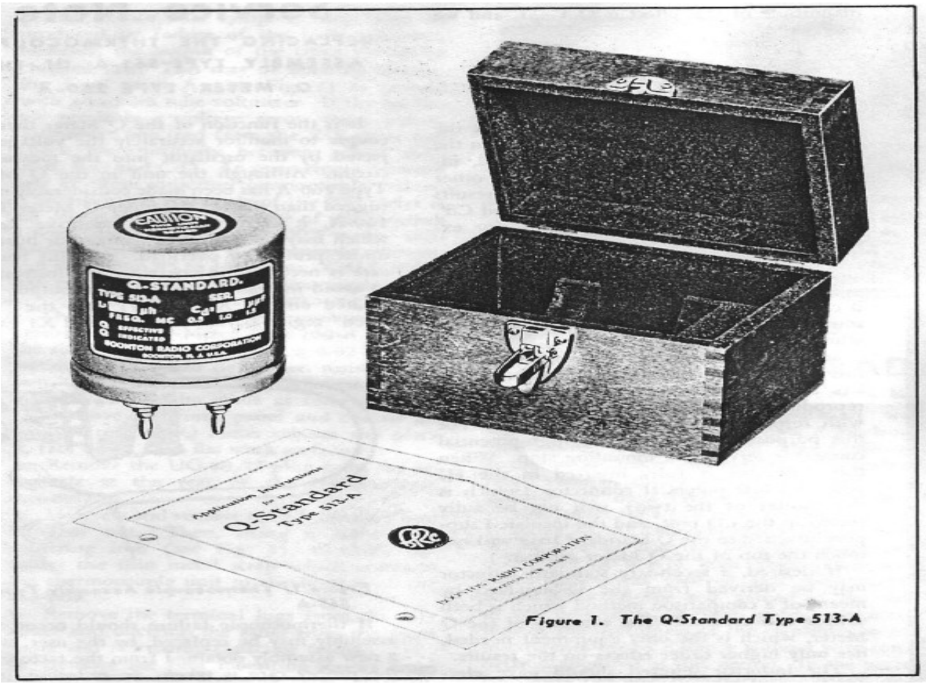


Figure 1. The Q-Standard Type 513-A

Basic Formulas Involving Q

A. TWO-TERMINAL IMPEDANCE

FORMULAS RELATING EQUIVALENT SERIES AND PARALLEL COMPONENTS

$$Q = \frac{X_s}{R_s} = \frac{\omega L_s}{R_s} = \frac{1}{\omega C_s R_s} = \frac{R_p}{X_p} = \frac{R_p}{\omega L_p} = R_p \omega C_p$$

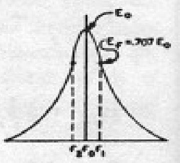
General Formula	Q greater than 10	Q less than 0.1	General Formula	Q greater than 10	Q less than 0.1
$R_s = \frac{R_p}{1+Q^2}$	$R_s = \frac{R_p}{Q^2}$	$R_s = R_p$	$R_p = R_s(1+Q^2)$	$R_p = R_s Q^2$	$R_p = R_s$
$X_s = X_p \frac{Q^2}{1+Q^2}$	$X_s = X_p$	$X_s = X_p Q^2$	$X_p = X_s \frac{1+Q^2}{Q^2}$	$X_p = X_s$	$X_p = \frac{X_s}{Q^2}$
$L_s = L_p \frac{Q^2}{1+Q^2}$	$L_s = L_p$	$L_s = L_p Q^2$	$L_p = L_s \frac{1+Q^2}{Q^2}$	$L_p = L_s$	$L_p = \frac{L_s}{Q^2}$
$C_s = C_p \frac{1+Q^2}{Q^2}$	$C_s = C_p$	$C_s = \frac{C_p}{Q^2}$	$C_p = C_s \frac{Q^2}{1+Q^2}$	$C_p = C_s$	$C_p = C_s Q^2$

B. TUNED CIRCUIT

1. Selectivity

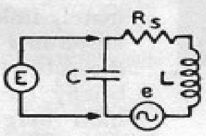
$$Q = \frac{f_0}{f_1 - f_2} = \frac{2C_0}{C_2 - C_1}$$

Where f_1 and f_2 are half-power points and C_0, C_1, and C_2 are capacitance values at f_0, f_2 and f_1 respectively.



2. Resonant Rise in Voltage $Q = \frac{E}{e}$

For relatively large R_s (low Q), $E = e\sqrt{1+Q^2}$

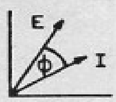


3. Power Dissipation

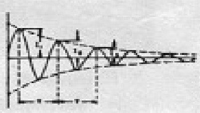
a. Power Factor = $\cos \phi$

$$= \frac{R}{\sqrt{R^2 + L^2\omega^2}} = \frac{1}{\sqrt{1+Q^2}}$$

and for inductors, $Q = \tan = \frac{\omega L}{\phi}$



b. Damped Oscillations $Q = \frac{\pi}{\delta}$, where δ is the logarithmic decrement.



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